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The similarity law between the temperature dependences of yield stress and microyield stress evaluated from internal friction

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Abstract

The amplitude-dependent internal friction gives the temperature dependence of the microyield stress $\sigma_\epsilon(T)$ at a very low (of the order of 10^{-10}–10^{-8}) value of the reversible dislocation strain. A similarity law (*i.e.* a proportionality of the macro- and microyield stresses) has been found for $\sigma_\epsilon(T)$ but not for other measures of microyield. The similarity law is not consistent with the conventional thermally activated approach. An approach based on the temperature dependence of the stress-induced plasticity is concluded to be responsible for the temperature dependences of both micro- and macroyield stresses.

1. Introduction

The amplitude-dependent internal friction (ADIF) in solids is a result of reversible plastic deformation occurring under cyclic loading and producing a static (in other words, frequency-independent) hysteresis [1]. Independence of frequency is a feature of a non-thermally activated mechanism. On the other hand, it is well known that the ADIF depends strongly on the temperature. The nature of this dependence was attributed to the influence of thermal fluctuations on the process of the breakaway of pinned dislocation lines from their pinning points [2–4]. Moreover, one can obtain [4, 5] from ADIF data the temperature dependence of the microyield stress in such a way that it reflects, in accordance with the breakaway theories [2–4], the microscopic parameters of dislocation–point defect interaction.

The origin of the temperature dependence of the yield stress σ_c in crystals is also considered to be a consequence of the thermally activated nature of plastic deformation [6–8].

It is of interest to compare the temperature dependence of the macroscopic yield stress $\sigma_c(T)$ with that of the microscopic yield stress. However, the microyield stress can be defined in various ways and different dependences $\sigma(T)$ may be obtained from the same ADIF data.

The aim of the present work is to discuss various definitions of microyield stress and to demonstrate that it is possible to define this stress in such a way that a similarity law between the temperature dependences of the macro- and microyield stresses is manifested. We argue that the similarity law cannot be unambiguously described within the framework of the conventional thermally activated approach.

2. General features of amplitude-dependent internal friction

Since the microyield stress is evaluated from ADIF data, let us first consider the general features of dislocation ADIF which are important for the manifestation of the similarity law.

(1) There is a proportionality between the amplitude-dependent decrement δ_h and the amplitude-dependent modulus defect $(\Delta E/E)_h$:

$$\delta_h = r\left(\frac{\Delta E}{E}\right)_h \tag{1}$$

where r is a coefficient of the order of unity [9–18].

(2) At low temperatures ($T<0.3T_m$, where T_m is the melting temperature) the ADIF can be expressed as

$$\delta_h = f_1(T)f_2(\sigma_o) \tag{2}$$

where σ_o is the vibrational stress amplitude related by Hooke's law to the elastic strain amplitude $\epsilon_o=\sigma_o/E$. Nowick [10] found that in copper at $213 \leqslant T \leqslant 306$ K $f_1 \propto \exp(-H/kT)$ and $f_2 \propto \sigma_o^n$ with $n=2$.

Power laws of amplitude dependences were observed in early experiments on steels (see ref. 1 for a review) and later in experiments on many other materials: copper [9, 10] and its alloys [11, 19], aluminium [20,

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21] and its alloys [22], ionic crystals [13, 23], etc. Thorough computer-controlled measurements of the ADIF carried out at $6 \leqslant T \leqslant 300$ K on b.c.c. (Mo [24]), f.c.c. (Al [14, 15], silicon bronze [16]) and h.c.p. (Mg [17]) metals and alloys and on MgO [18] crystals show that the separation (2) really does take place and f_2 may be well approximated by a power-type function. Thus eqn. (2) may be rewritten as

$$\delta_h = f_1(T)\sigma_o^n \tag{3}$$

The value of n generally lies within the range from $n=1$ or slightly lower to $n=4$ or slightly higher. It is sensitive to the impurity content (higher values of n are observed for concentrated solid solutions [11, 13, 19, 22]). From the data of Goto *et al.* [25] on polycrystalline copper it follows that n is higher in samples with a smaller grain size.

The functional form of $f_1(T)$ is still unknown for the majority of crystals. Baker and Carpenter [26] found that in tungsten at $80 \leqslant T \leqslant 370$ K f_1 may be approximated by an Arrhenius-type curve, *i.e.* $f_1 \propto \exp(-H/kT)$, as in Cu [10]. Saul and Bauer [12] observed a similar behaviour only at $180 \leqslant T \leqslant 300$ K in dilute Cu alloys, whereas within the temperature range from 5 to 300 K the experimental points agreed well with $f_1 \propto \exp(LT)$, where L is a constant independent of the amplitude.

Systematic investigations carried out on Al [14, 15] and high purity Mo [24] show that at low temperatures ($T<90$ K in pure Al and $T<150$–170 K in Al alloys and Mo) the dependence of the decrement on σ_o and T may be well approximated as $\delta_h = A\sigma_o^n \exp(LT)$, *i.e.* $f_1 = A \exp(LT)$, where A is some coefficient.

3. Microyield stress evaluated from internal friction data

It is possible to define a critical stress with a clear physical meaning, *e.g.* σ_E, the limit of elasticity which corresponds to the appearance of the first closed loop on the stress–strain diagram during cyclic loading [27], or σ_1, the critical amplitude beyond which the decrement increases [28, 29]. However, a precise registration of the onset of both macroyielding and microyielding is practically impossible [27]. To avoid this difficulty, a number of "conditional" stresses were defined. One can choose an *a priori* condition and then consider the changes in stress with *e.g.* temperature under the given condition. Let us consider three definitions of microyield stress with different *a priori* criteria.

(1) Roberts and Hartman [30] considered the temperature dependence of the vibrational stress σ_w at a constant value of energy losses ΔW. Since the decrement of vibrations is defined by $\delta = \Delta W / E\epsilon_o^2$), the condition $\Delta W=$ const. is equivalent in ADIF experiments to $\delta_h \epsilon_o^2 =$ const.

(2) Schwarz and Granato [4] considered the vibrational stress amplitude σ_δ at $\delta_h =$ const. as the microyield stress. According to the theories of ADIF [3, 4] based on the Köhler–Granato–Lücke model [31], the stress σ_δ ensures a constant number of dislocation segments contributing to the ADIF.

(3) In refs. 32–34 the microyield stress σ_ϵ was defined as the stress at a constant level of dislocation strain ϵ_d. The value of ϵ_d can be evaluated from ADIF data in at least two ways. The first method is to use empirical relation (1) taking into account the generally accepted expression for the dislocation modulus defect at $\epsilon_d \ll \epsilon_o$ [35]:

$$\frac{\Delta E}{E} = \frac{\epsilon_d}{\epsilon_o} \tag{4}$$

Then the condition $\epsilon_d =$ const. is equivalent to $\delta_h \epsilon_o =$ const. [32, 34].

The second method [19, 25, 33] is based on Asano's theory [36], which allows one to obtain $\epsilon_d(\sigma)$ curves from $\delta_h(\sigma_o)$ dependences and vice versa. If eqn. (3) is valid, then these two methods give the same dependence $\sigma_\epsilon(T)$; however, for the latter ϵ_d is smaller by a factor of 2^n than for the former [37].

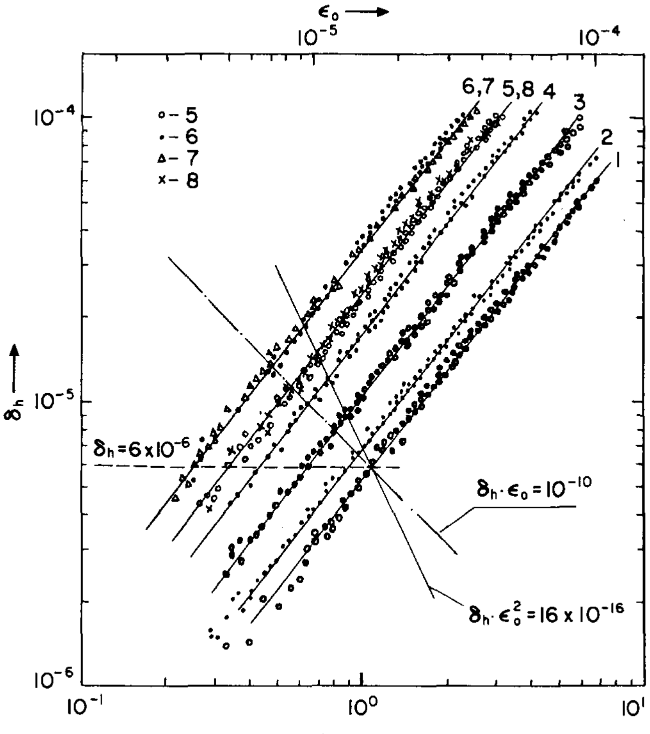

Fig. 1. Amplitude-dependent part of decrement at temperatures $T=6.5$ (1), 26 (2), 60 (3), 100 (4), 140 (5), 197 (6), 227 (7) and 247 K (8) for Al–Si–Fe alloy [15, 16]. The intersections of the $\delta_h(T)$ curves with the three lines $\delta_h \epsilon_o^2 =$ const., $\delta_h \epsilon_o =$ const. and $\delta_h =$ const. give $\sigma_w(T)$, $\sigma_\epsilon(T)$ and $\sigma_\delta(T)$ respectively.

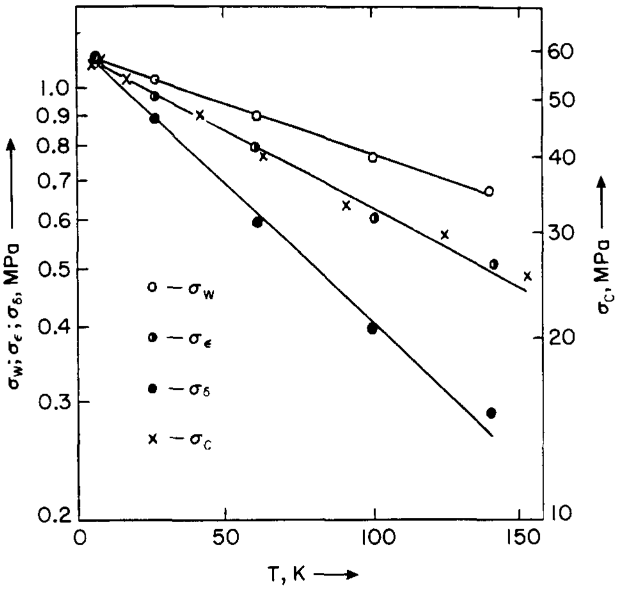

Fig. 2. Temperature dependences of σ_w, σ_δ, σ_ϵ (data from Fig. 1) and σ_c.

In fact, the macroscopic flow stress is defined as the stress σ_c corresponding to a certain value of irreversible plastic strain (as a rule, not less than 10^{-4}–10^{-3}). It is reasonable to apply the same approach in the microscopic region and to choose σ_ϵ as the microyield stress.

Lebedev and Kustov have reported [34] that it is possible to evaluate $\sigma_\epsilon(T)$ at very low ϵ_d values (of the order of 10^{-10}–10^{-8}), when the dislocation strain is completely reversible. Moreover, the dependences $\sigma_\epsilon(T)$ were found to be proportional to the temperature dependences of the macroscopic yield stress $\sigma_c(T)$ at $4.2 \leqslant T \leqslant 300$ K for a large variety of crystals. This similarity law, $\sigma_\epsilon(T) \propto \sigma_c(T)$, is strongly supported by recent results obtained on single crystals of high purity Mo [24] and of the alloys Al–0.66wt.%Si–0.22wt.%Fe [14, 15] and Cu-Ni [19].

As an example, Figs. 1 and 2 demonstrate the application of the three above-mentioned approaches to the processing of the experimental data from refs. 14 and 15 obtained on Al–Si–Fe single crystals. From Fig. 1, where $\delta_h(\epsilon_o)$ curves measured at various temperatures at a frequency near 100 kHz are plotted on a double-logarithmic scale, the validity of eqn. (3) is clearly evident. Intersections of the $\delta_h(\epsilon_o)$ curves with three lines give the temperature dependences of the three microyield stresses $\sigma_w(T)$, $\sigma_\delta(T)$ and $\sigma_\epsilon(T)$. Experimental points for these dependences are plotted in Fig. 2. The points for the yield stress $\sigma_c(T)$ are also shown and it is clear that the similarity law between the temperature dependences of the macroyield and microyield stresses is valid for σ_ϵ only.

4. Discussion

Thermally activated [6–8] and/or stress-induced [38–40] plasticity can be considered as the basis of the similarity law.

If the plastic strain rate $\dot\epsilon_d$ obeys the Arrhenius-type equation

$$\dot\epsilon_d = \dot\epsilon_a \exp\left(-\frac{H(\sigma)}{kT}\right) \qquad (5)$$

where $\dot\epsilon_a$ is some coefficient and $H(\sigma)$ is the stress-dependent activation enthalpy, then $\sigma(T)$ may be derived by solving (5) under the condition $\dot\epsilon_d = \text{const}$. In this case $\sigma(T)$ is obviously influenced by the strain rate and is determined by the functional form of $H(\sigma)$ characteristic of the specific mechanism of a dislocation overcoming a barrier.

The main feature of stress-induced plasticity is the existence of a functional relation between strain and stress (not between the strain rate and the stress as in the case of an Arrhenius-type equation). The dislocation strain follows from eqns. (1), (3) and (4) as

$$\epsilon_d = \frac{f_1(T)\sigma_o^{n+1}}{rE} \qquad (6)$$

which in fact gives the equation of stress-induced plasticity.

If the similarity law is due to the thermally activated overcoming of barriers by dislocations, then not only should the barriers be of the same nature and have the same $H(\sigma)$ dependence in the micro- and macroplastic ranges, but the average dislocation velocities should also be the same under static loading and under vibrational stresses at frequencies up to 100 kHz.

During ADIF experiments the dislocation strain rate $\dot\epsilon_d$ is proportional to ϵ_d. Obviously, if the frequency f is fixed, then $\dot\epsilon_d \propto f\epsilon_d$ [41].

Within the range of validity of the separation (2) the temperature trend of σ_ϵ, *i.e.* $\sigma_\epsilon(T)$ in relative units, is independent of the level of ϵ_d and hence independent of $\dot\epsilon_d$. The similarity law $\sigma_\epsilon(T) \propto \sigma_c(T)$ is reported to hold well for σ_ϵ evaluated from the ADIF measured at both 100 kHz [34] and 1 kHz [19]. These facts imply that the dependences $\sigma_\epsilon(T)$ and $\sigma_c(T)$ are not in agreement with eqn. (5). Thus some other mechanism must be responsible for the temperature dependence.

The physical basis for such a mechanism probably lies in an alternative approach to thermal activation. In contrast with the generally accepted concept [6–8], one can suggest that not the thermal fluctuations but the thermal oscillations of atoms are the reason for the decrease in the force necessary for a dislocation to overcome a barrier. This idea was suggested by Leibfried [42] and subsequently developed by Saul and

Bauer [12]. However, the problem requires further theoretical studies.

Butt and Chaudhry [43] have reported that for a large variety of f.c.c. and b.c.c. metals there is a functional relationship between the macroscopic yield stress and the value of the mean-square amplitude of atomic vibrations, $\langle u^2 \rangle$.

It should be noted that within the $\langle u^2 \rangle$ approach the temperature dependence of the stress is not related to the strain rate. Thus the hypothesis [34] on the relation between the nature of $\sigma_c(T)$ and stress-induced plasticity which follows from the similarity law is strongly supported. The origin of the similarity law might be the general dependence of $\langle u^2 \rangle$ on T, which is not related to a specific mechanism of dislocation–barrier interaction. One can suggest that $f_1(T)$ in eqns. (2), (3), and (6) plays essentially the same role as the Debye–Waller factor in crystal structure analysis.

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References

1 N.N. Davidenkov, *Zh. Tech. Fiz., 8* (1938) 483.
2 K. Lücke, A.V. Granato and L.J. Teutonico, *J. Appl. Phys., 39* (1968) 5181.
3 V.L. Indenbom and V.M. Chernov, *Phys. Status Solidi A, 14* (1972) 347.
4 R.B. Schwarz and A.V. Granato, *Phys. Rev. Lett., 38* (1975) 1174.
5 B.K. Kardashev, *Fiz. Tverd. Tela, 19* (1977) 2490 (Engl. transl. *Sov. Phys.-Solid State, 19* (1977) 1459).
6 A. Seeger, *Philos. Mag., 45* (1954) 771; *46* (1955) 1194.
7 J. Friedel, *Dislocations,* Pergamon, Oxford, 1964.
8 G. Schoeck, in F.R.N. Nabarro (ed.), *Dislocations in Solids,* Vol. 3, North-Holland, Amsterdam, 1980, p. 63.
9 T.A. Read, *Trans. AIME, 143* (1941) 30.
10 A.S. Nowick, *Phys. Rev., 80* (1950) 249.
11 S. Takahashi, *J. Phys. Soc. Jpn., 11* (1956) 1253.
12 R.H. Saul and C.L. Bauer, *J. Appl. Phys., 39* (1968) 1469.
13 S.P. Nikanorov and B.K. Kardashev, *Elasticity and Dislocation Anelasticity of Crystals,* Nauka, Moscow, 1985, p. 145 (in Russian).
14 A.B. Lebedev, *Fiz. Tverd. Tela, 34* (1992) 1889 (Engl. transl. *Sov. Phys-Solid State, 34* (1992) 1008).
15 A.B. Lebedev and V.I. Ivanov, *Mater. Sci. Forum, 119–121* (1993) 245.
16 S. Pileski, V.I. Ivanov and A.B. Lebedev, *Phys. Status Solidi A, 119* (1990) 87.
17 V.I. Ivanov, A.B. Lebedev, B.K. Kardashev and S.P. Nikanorov, in F.N. Tavadze (ed.), *Internal Friction and Fine Structure of Metals and Inorganic Materials,* Nauka, Moscow, 1985, p. 102 (in Russian).
18 B.K. Kardashev, S.B. Kustov, A.B. Lebedev, G.V. Berezhkova, P.P. Perstnev, F. Appel and U. Messerschmidt, *Phys. Status Solidi A, 91* (1985) 79.
19 Y. Nishino, Y. Okada and S. Asano, *Phys. Status Solidi A, 129* (1992) 409.
20 D. Gelli, *J. Appl. Phys., 33* (1962) 1547.
21 A.V. Kharitonov, *Akust. Zh., 9* (1963) 80 (Engl. transl. *Sov. Phys.-Acoust., 9* (1963) 63).
22 D. Gelli, *Phys. Status Solidi, 12* (1965) 829.
23 B.K. Kardashev, S.P. Nikanorov and O.A. Voinova, *Phys. Status Solidi A, 12* (1972) 375.
24 A.B. Lebedev, B.K. Kardashev, U. Hofmann, H.-J. Kaufmann and D. Schulze, *Cryst. Res. Technol., 24* (1989) 1143.
25 H. Goto, Y. Nishino and S. Asano, *J. Jpn. Inst. Met., 55* (1991) 848.
26 G.S. Baker and S.H. Carpenter, *J. Appl. Phys., 38* (1967) 3557.
27 N. Brown, in C.J. McMahon Jr. (ed.), *Microplasticity,* Wiley, New York, 1968, p. 45.
28 S. Takahashi, *J. Appl. Phys., 23* (1952) 866.
29 M.A. Krishtal and S.A. Golovin, *Internal Friction and Structure of Metals,* Metallurgiya, Moscow, 1976, p. 235 (in Russian).
30 J.M. Roberts and D.E. Hartman, *Trans. AIME, 230* (1964) 1125.
31 A. Granato and K. Lücke, *J. Appl. Phys., 27* (1956) 583.
32 V.A. Chelnokov and N.L. Kuz'min, *Fiz. Tverd. Tela, 24* (1982) 3186 (Engl. transl. *Sov. Phys,-Solid State, 24* (1982) 1796).
33 H. Narai and S. Asano, *J. Jpn. Inst. Met., 46* (1982) 253.
34 A.B. Lebedev and S.B. Kustov, *Phys. Status Solidi A, 116* (1989) 645.
35 A.S. Nowick and B.S. Berry, *Anelastic Relaxation in Crystalline Solids,* Academic, New York, 1972, p. 359.
36 S. Asano, *J. Phys. Soc. Jpn., 29* (1970) 952.
37 A.B. Lebedev and S.B. Kustov, *Phys. Status Solidi A, 136* (1993) K85.
38 O.V. Klyavin, V.A. Likhachev, A.V. Nikiforov and V.A. Shvetsova, *Vest. Leningrad. Univ. Ser. Math., Mech., Astron.,* (7) (1978) 75.
39 V.A. Likhachev and V.A. Shvetsova, *Probl. Prochnosti,* (11) (1984) 57.
40 T.H. Alden, *Metall. Trans. A, 16* (1985) 375.
41 G.S. Baker, *J. Appl. Phys., 33* (1962) 1730.
42 G. Leibfred, in *Dislocations and Mechanical Properties of Crystals,* Wiley, New York, 1957, p. 495.
43 M.Z. Butt and S.A. Chaudhry, *Phys. Rev. B, 47* (1993) 8418.